Electronics Night: Basic AC Analysis

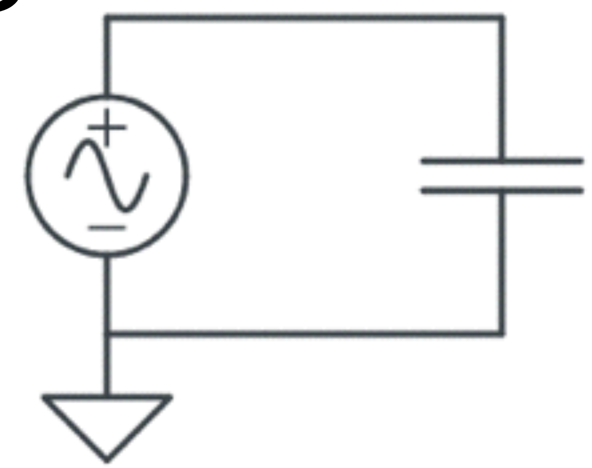
(slides will **not** be a regular feature)

Capacitor: current leads voltage by 90 degrees

$$V(t) = \sqrt{2}\sin(2\pi t)\,\mathrm{V}$$

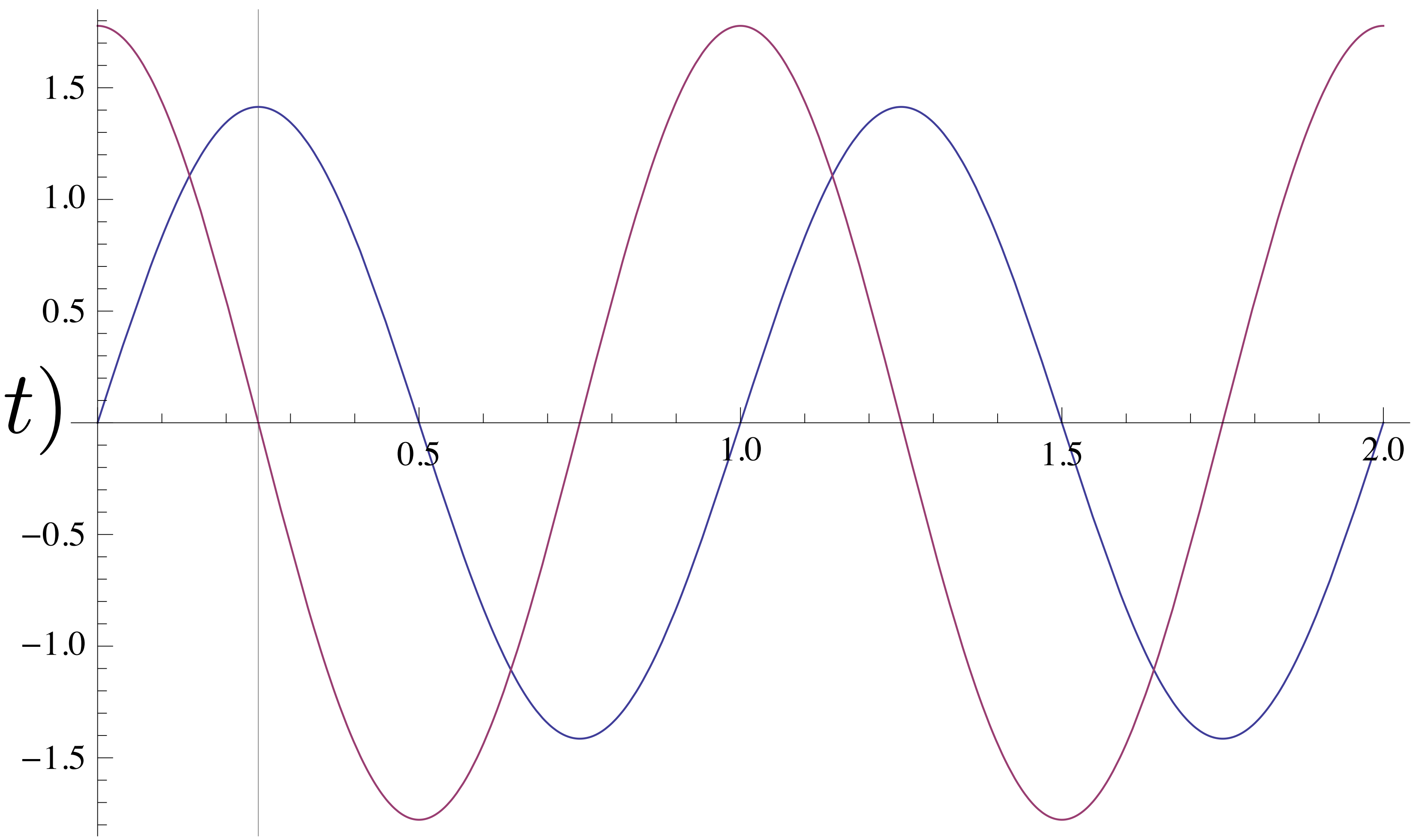
(1V-rms)

C=0.2F

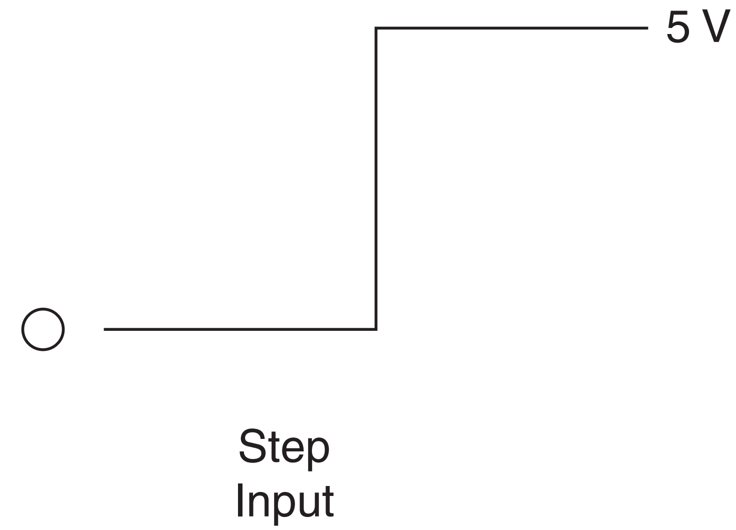


$$I(t) = C\frac{dV(t)}{dt}$$

$$\frac{dV(t)}{dt} = 2\sqrt{2}\pi\cos(2\pi t)$$



A capacitor resists **changes in voltage** by storing energy in an electric field



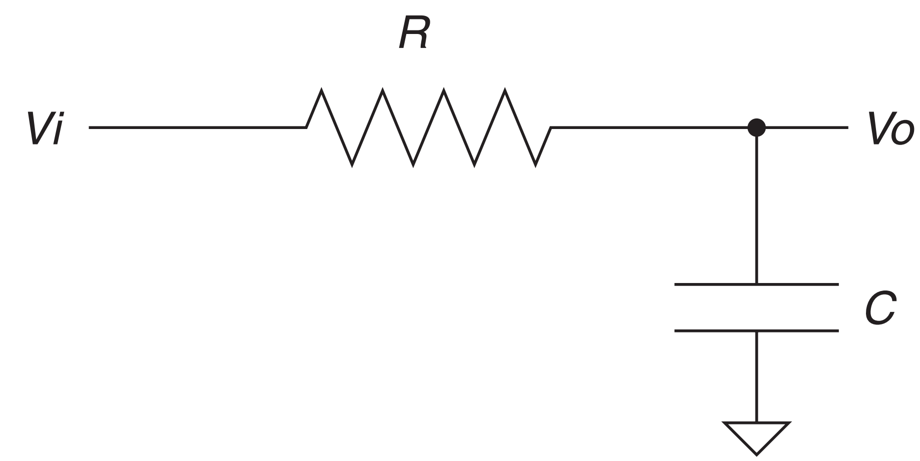
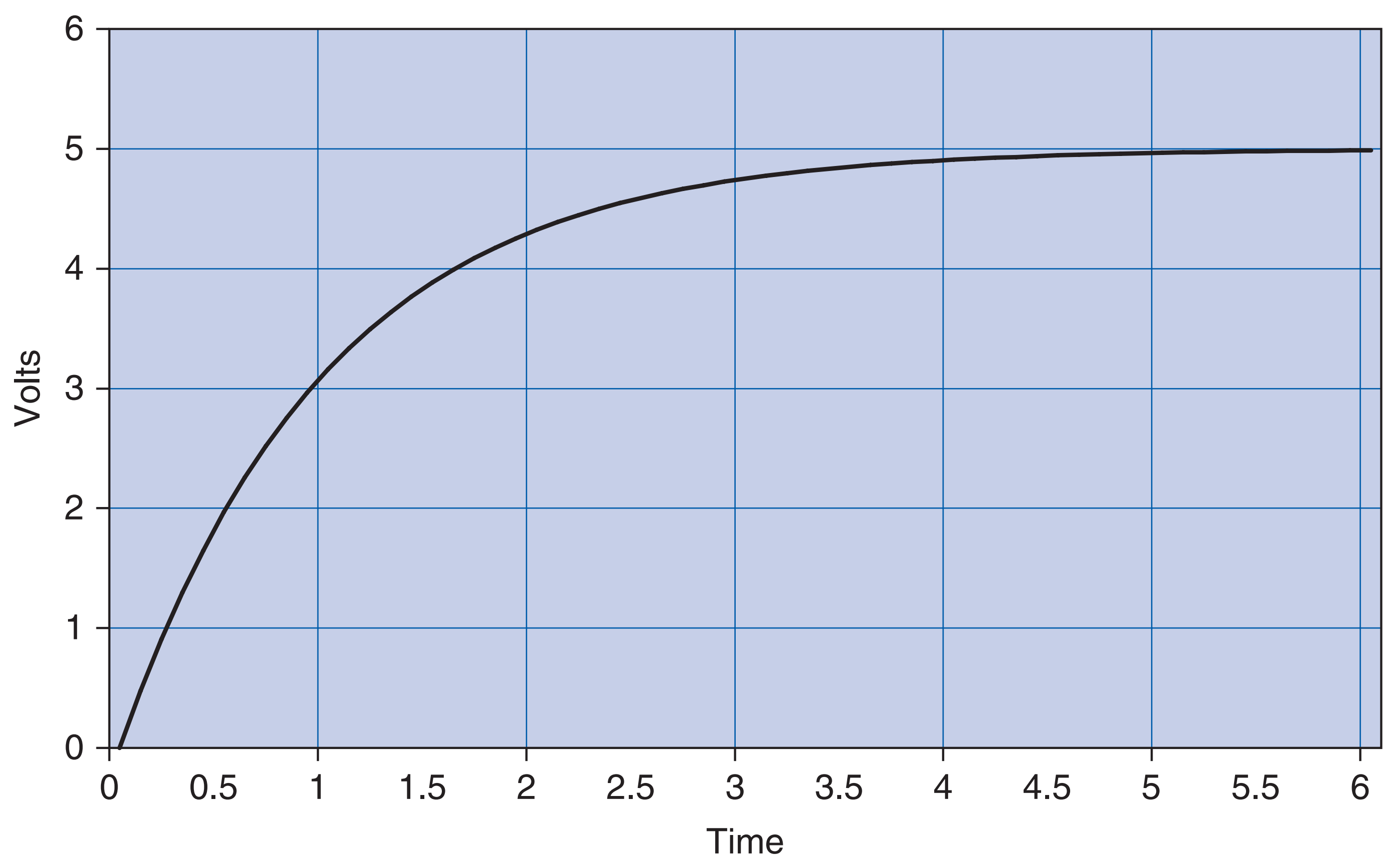


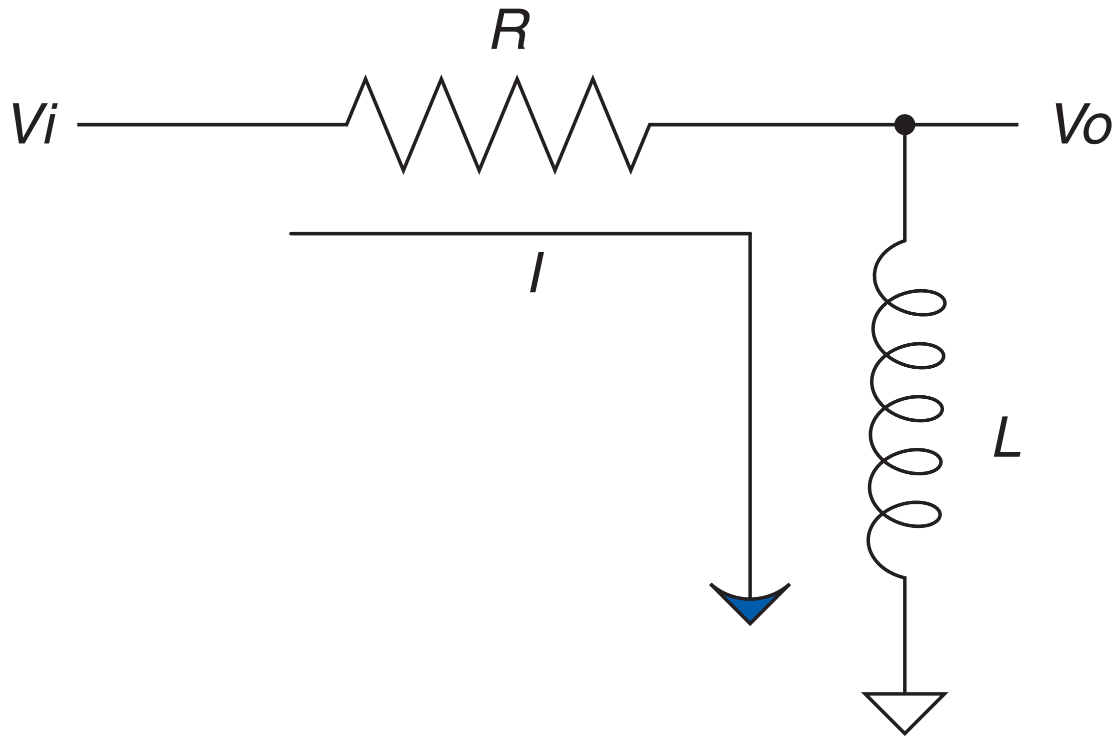
FIGURE 2.6
Step input is applied to a simple RC circuit.

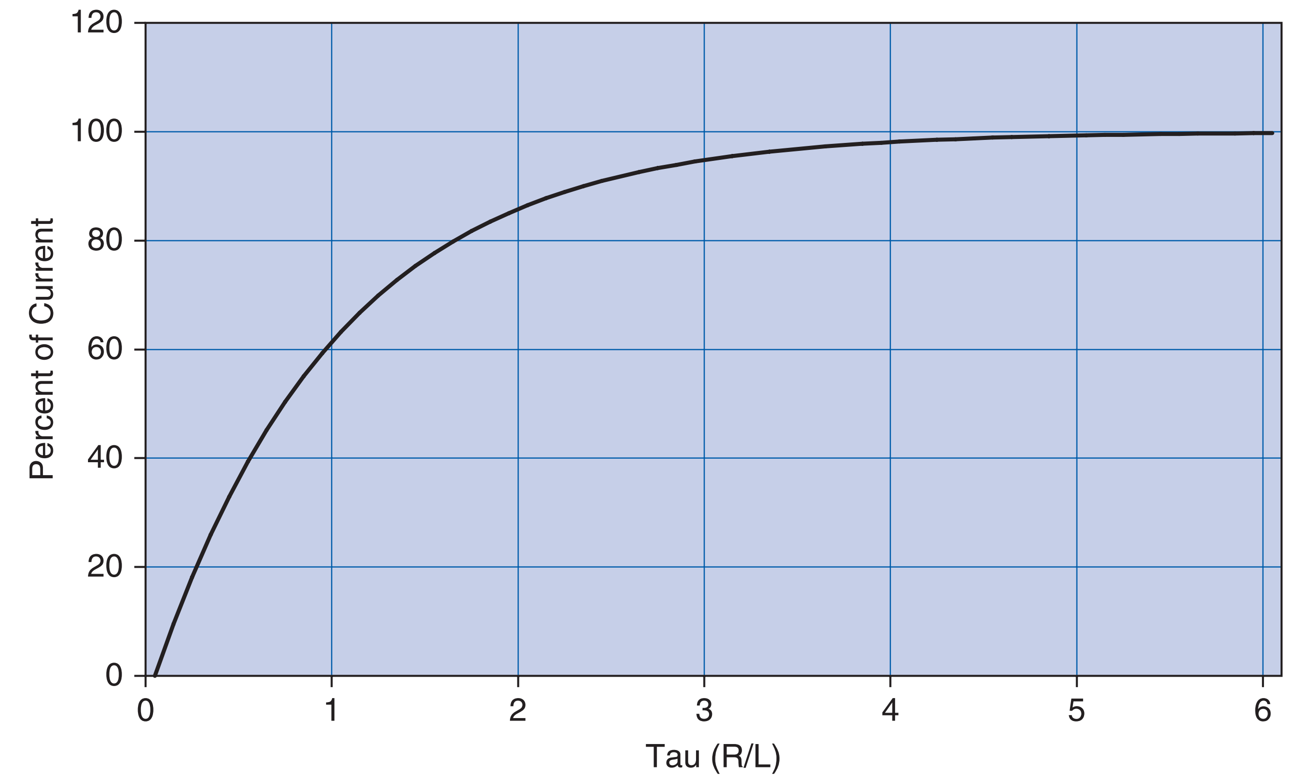


$$Vo = Vi\left(1 - e^{-\frac{\tau}{rc}}\right)$$

Reaches 63% of steady state voltage in one time constant

An inductor resists **changes in current** by storing energy in a magnetic field





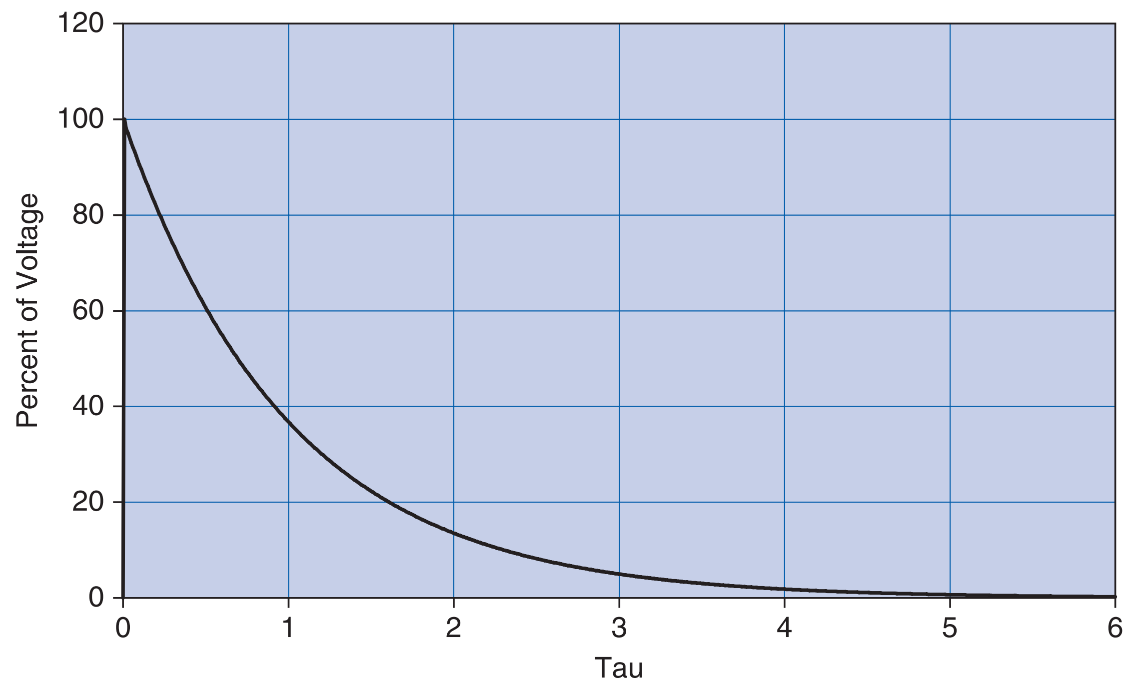


FIGURE 2.12
Voltage change in percent over time in tau.

Reactance

$$X_c = \frac{1}{2\pi f C}$$

$$X_L = 2\pi f L$$

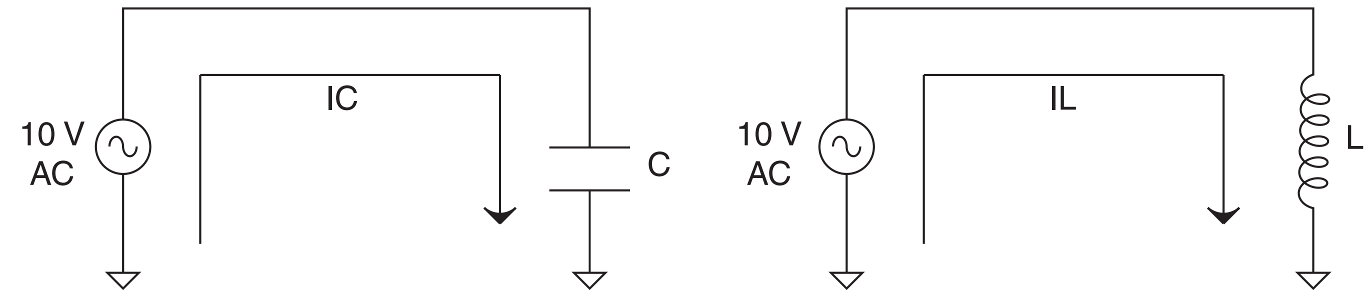


FIGURE 2.30
AC source hooked up to cap and inductor.

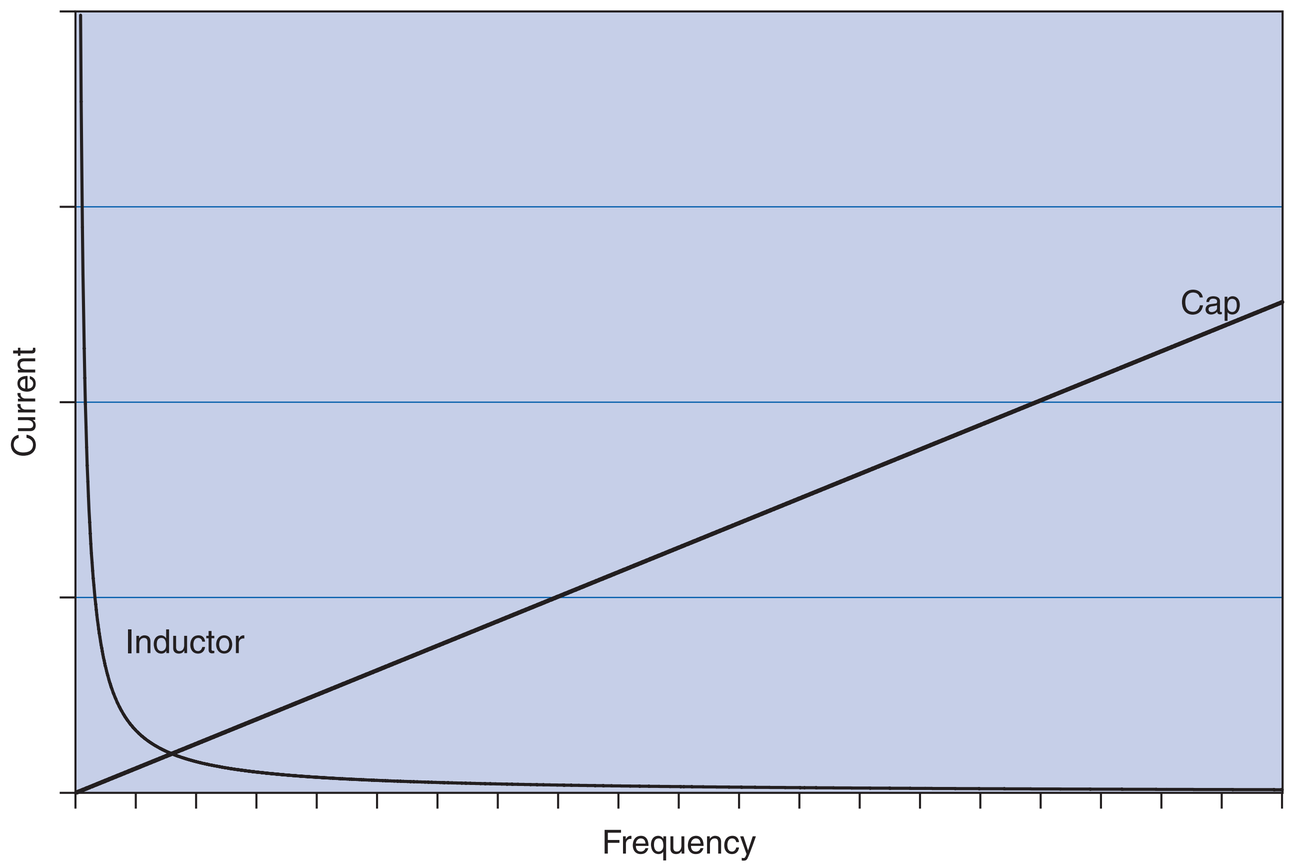
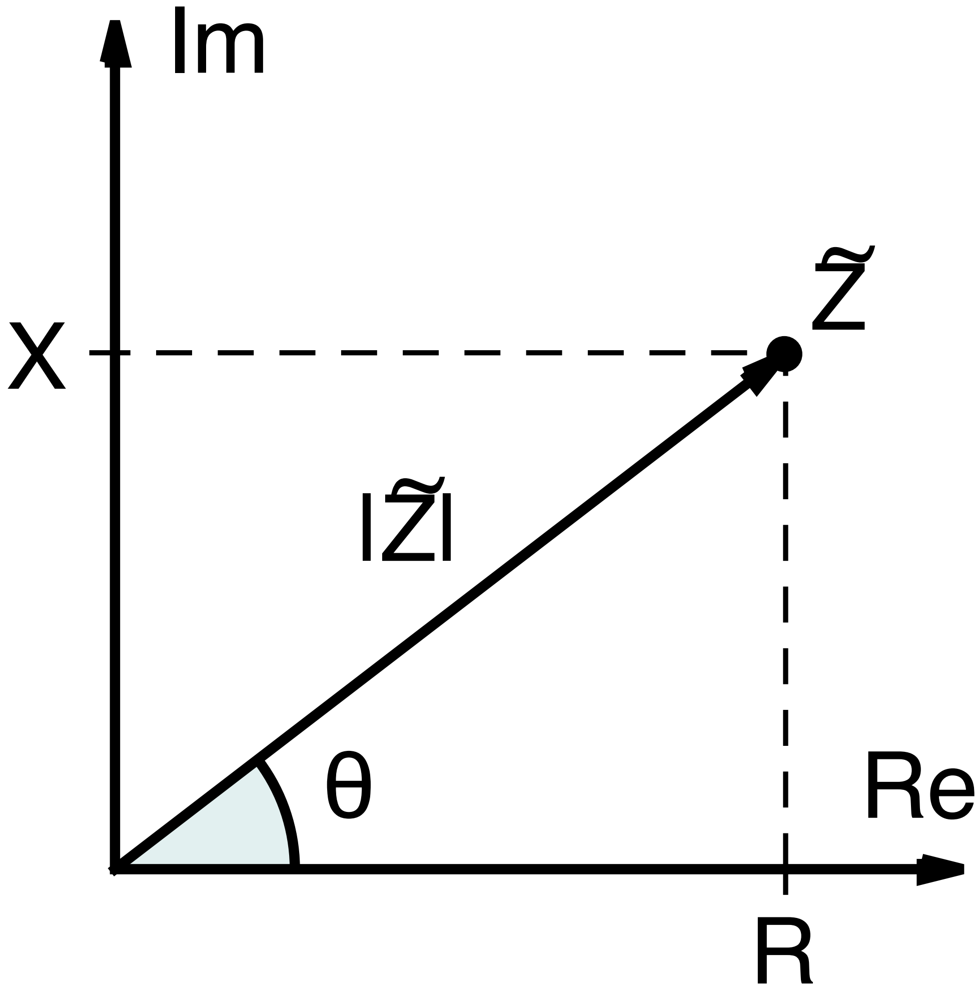


FIGURE 2.31
Graph of current over frequency for a cap and an inductor.

Impedance



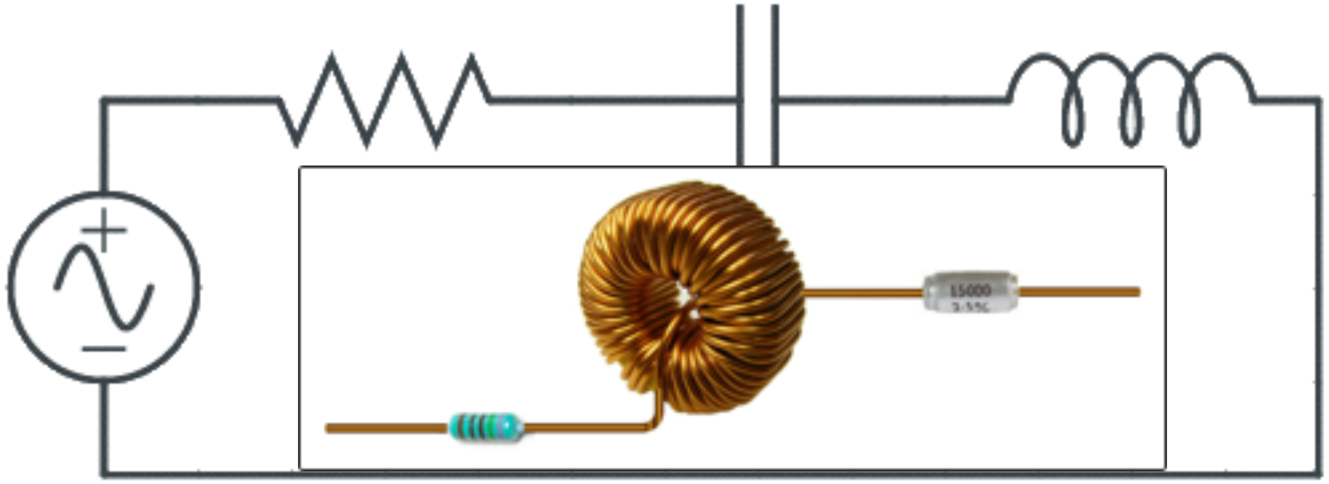
Real part: **resistance**

Imaginary part: **reactance**

Magnitude: **voltage/current**

Angle: voltage **phase shift**

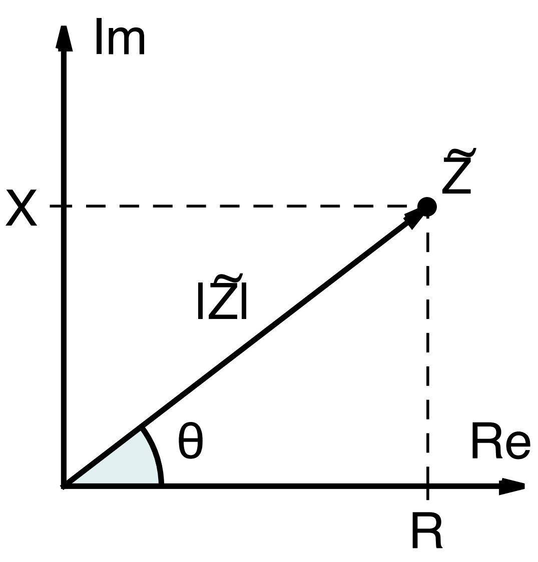
This is essentially **frequency domain analysis**



$$X_C = 500\,\Omega, \quad R = 1\,\mathrm{k}\Omega, \quad X_L = 250\,\Omega$$

$$X = X_L - X_C = -250\,\Omega$$

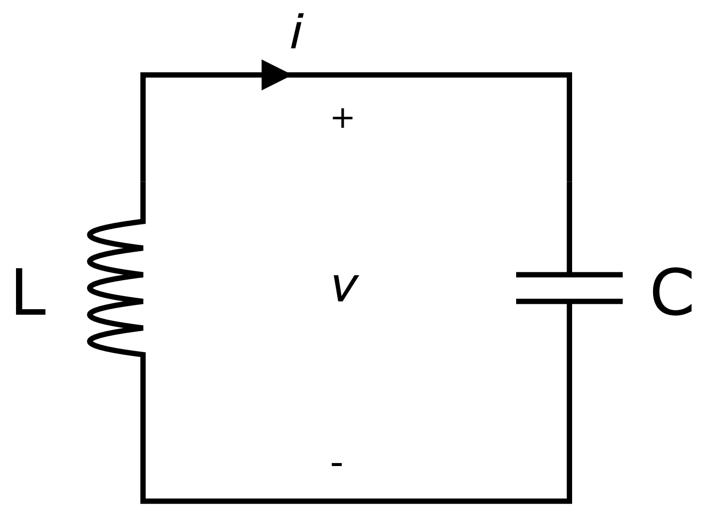
$$\theta = -\sin^{-1}(250/1000) = -14^\circ$$



Voltage lagging current 14 degrees

Depends on frequency!

Resonance



At the resonant frequency, capacitive reactance equals inductive reactance

$$2\pi f L = \frac{1}{2\pi f C}$$

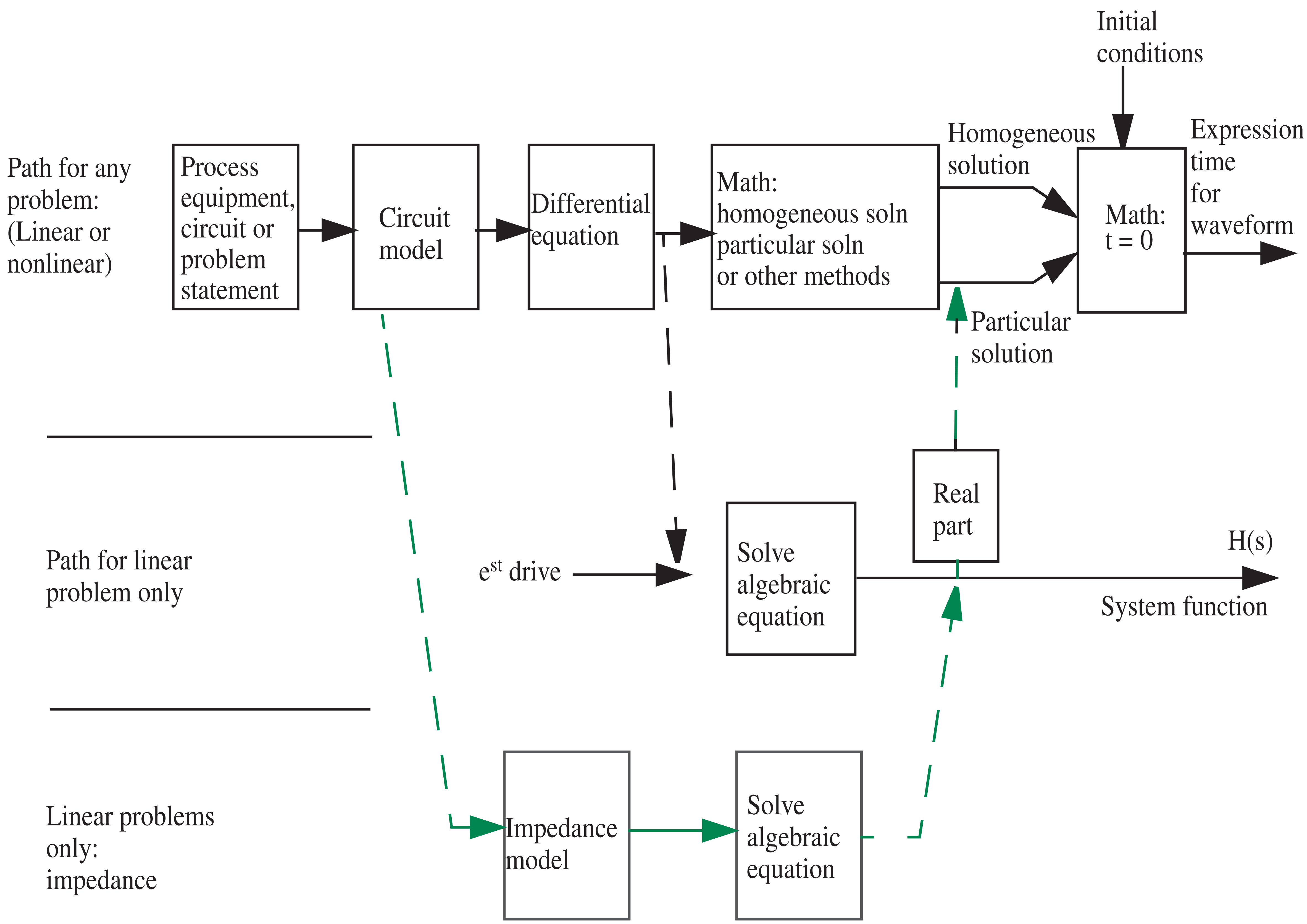
$$f = \frac{1}{2\pi\sqrt{LC}}$$

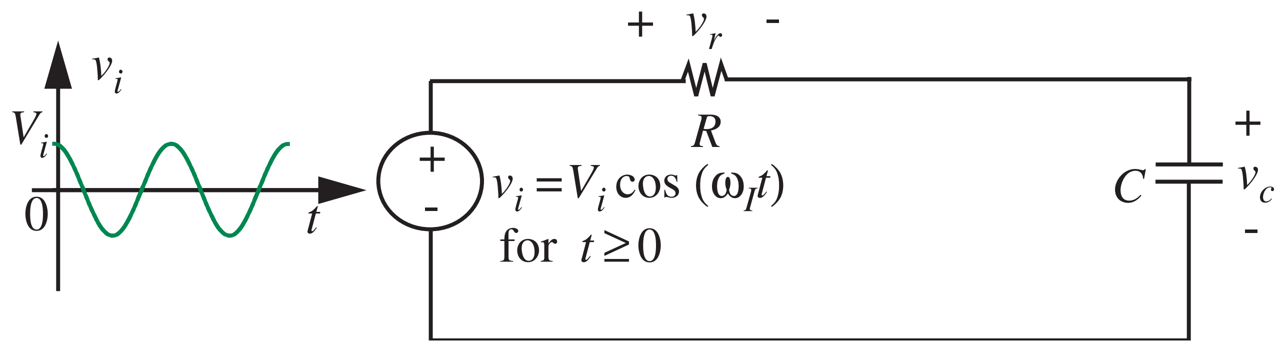
Example

The resonant frequency of a series RLC circuit if R is 22 ohms, L is 50 microhenrys and C is 40 picofarads is 3.56 MHz. (E5A14)

$f = 1/2\pi\sqrt{(LC)} = 1/6.28\text{x}\sqrt{(50\times10^{-6} \text{ x } 40\times10^{-12})} = 1/2.8\times10^{-7} =$ **3.56 MHz**

General solution method





$$v_i = V_i \cos(\omega_1 t) \quad \text{for} \quad t \geq 0,$$

$$v_i = v_c + RC\frac{dv_c}{dt}.$$

Homogenous solution:

$$RC\frac{dv_{ch}}{dt} + v_{ch} = 0. \qquad v_{ch} = K_1 e^{-t/RC}$$

Particular solution:

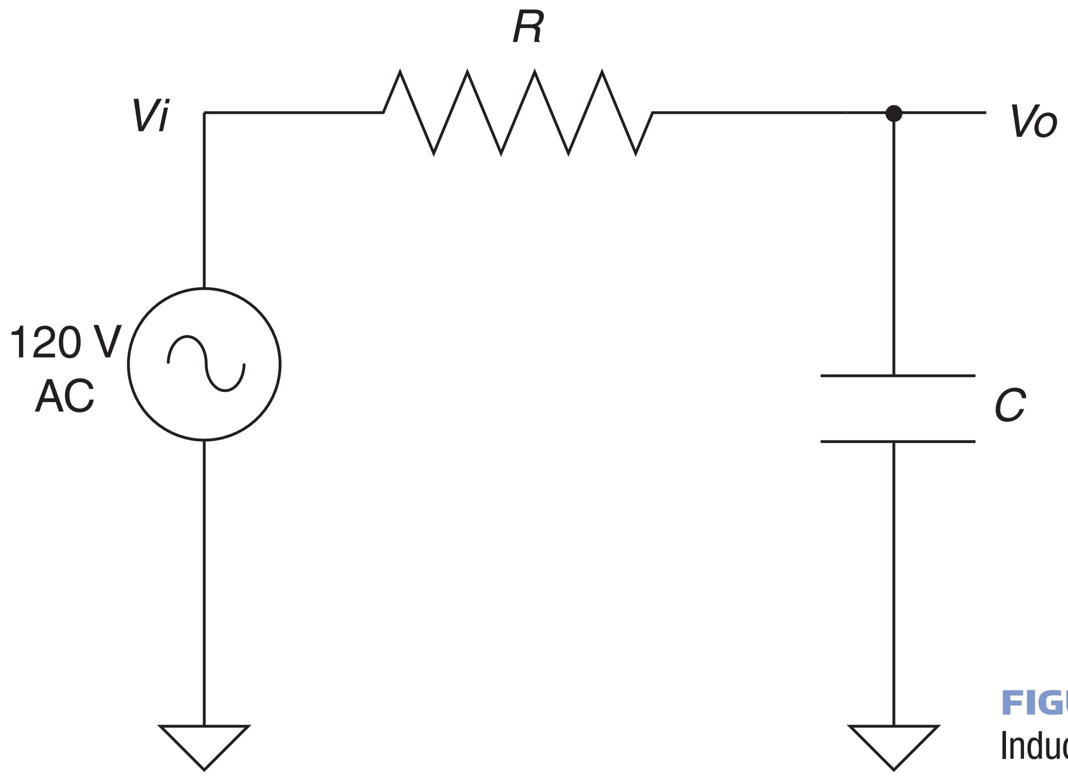
$$\tilde{v_{cp}} = \frac{V_i}{1 + j\omega_1 RC} e^{j\omega_1 t}. \qquad e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$$

Complete solution (steady state):

$$v_c = \frac{V_i}{\sqrt{1 + (\omega_1 RC)^2}} \cos(\omega_1 t + \Phi)$$

$$v_c \simeq \frac{V_i}{\omega_1 RC} \cos(\omega_1 t - 90°)$$

Filters



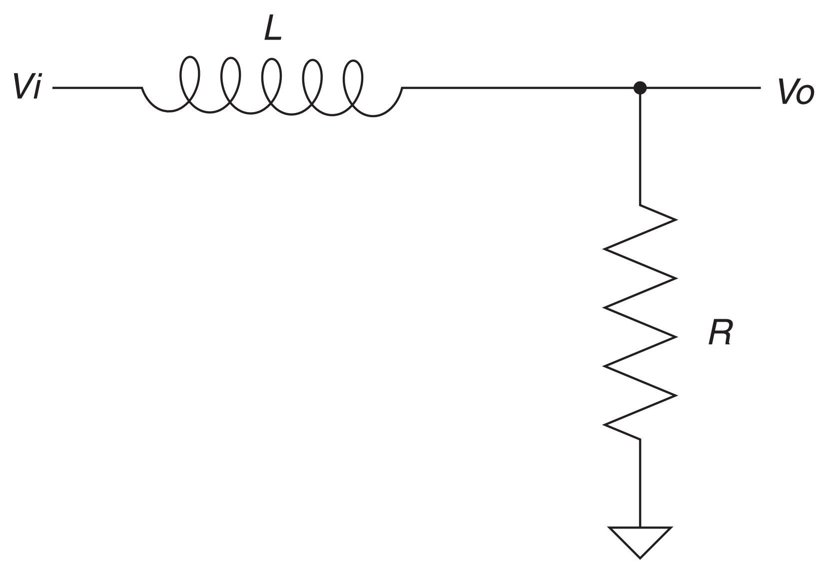


FIGURE 2.33
Inductor-based low-pass filter.

FIGURE 2.32
Cap-based low-pass filter.

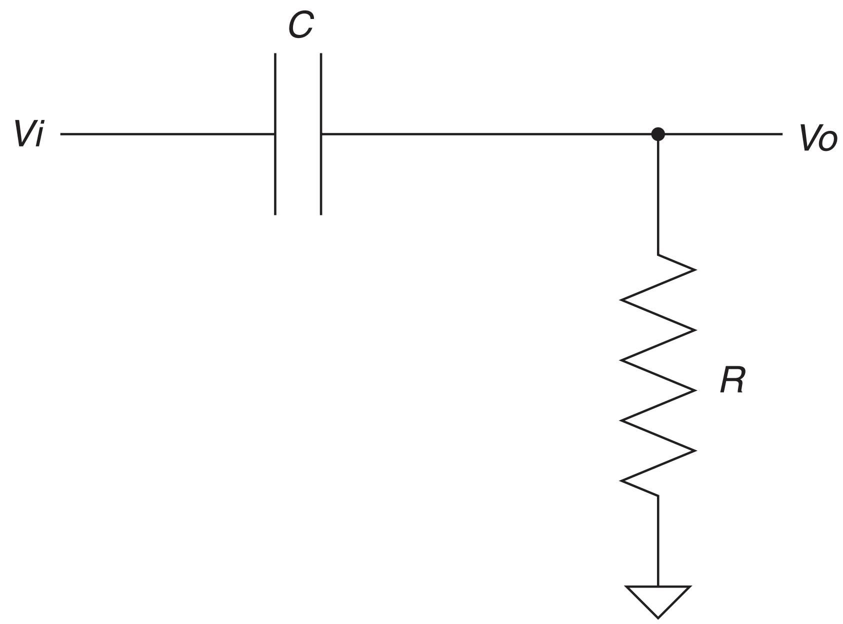
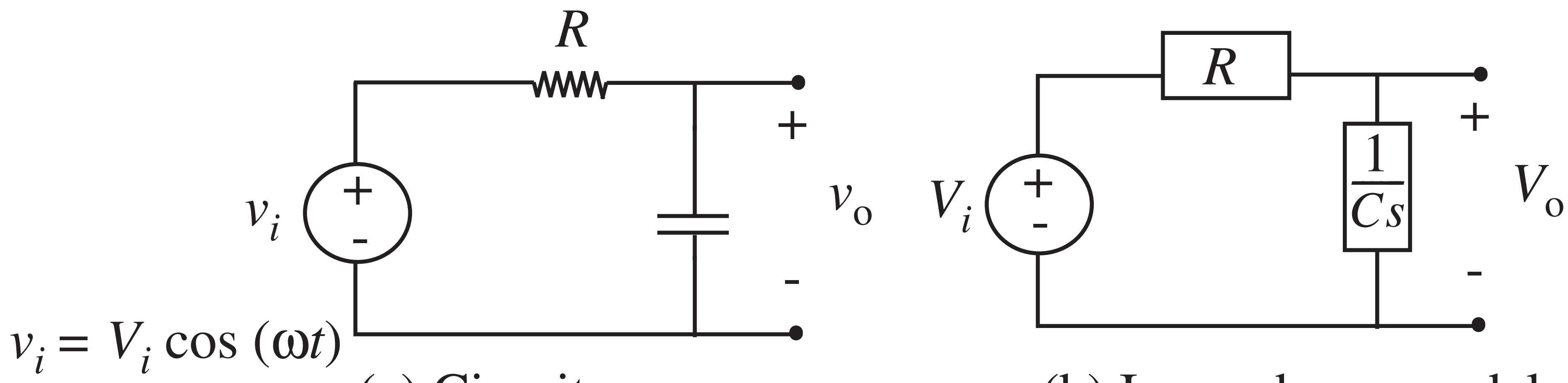


FIGURE 2.34
Cap-based high-pass filter.

Low pass RC filter



(a) Circuit (b) Impendance model

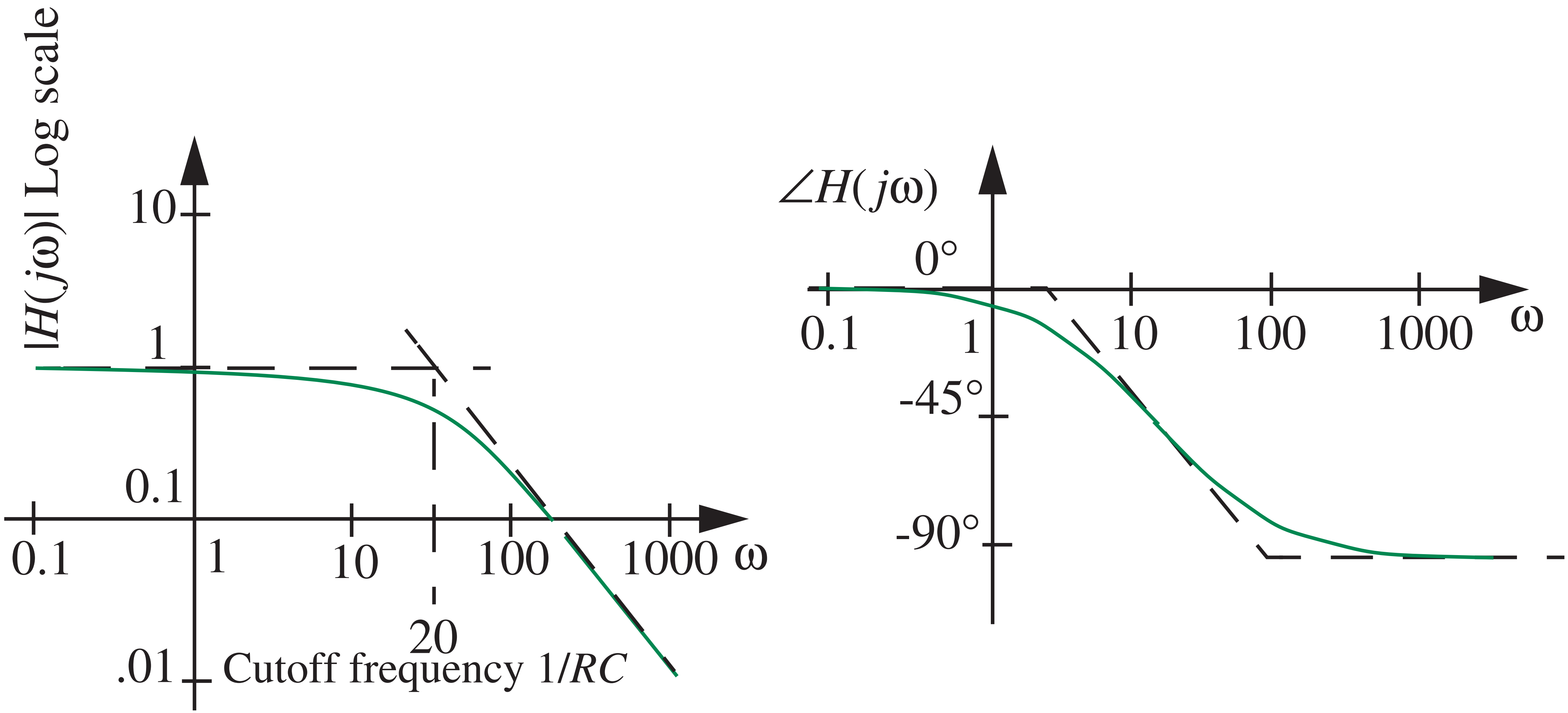
$(s \text{ is shorthand for } 2i\pi f)$

Voltage divider: $$V_o = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} V_i = \frac{1/RC}{s + 1/RC} V_i$$

Gain: $$H(2i\pi f) = \frac{V_o}{V_i} = \frac{1/RC}{2i\pi f + 1/RC}$$

What happens at low frequencies?

What happens at high frequencies?



|H(jω)| Log scale
10
1
0.1
.01
0.1
1
10
100
1000
ω
20
Cutoff frequency 1/RC
∠H(jω)
0°
-45°
-90°
0.1
1
10
100
1000
ω